On the Polarization Properties of the Far-Zone Radiation Fields of Primary and Secondary Electromagnetic Sources

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The far-zone field of a transmitting antenna (primary source) resembles the far-zone field by a scattered (secondary source) in that each of the far-zone fields is a TEM wave travelling radially outward from a body of finite spatial extent. Choosing a spherical coordinate system centered on the body, (see Fig. 1) we note that in the far-zone of the body the electric vector has the form

$$\underline{E} = \mathrm{Re}[\underline{e}_\theta F_\theta(\theta,\phi) + \underline{e}_\phi F_\phi(\theta,\phi)] \exp[i(kr - \omega t)]/4\pi r$$

where $\underline{e}_\theta$, $\underline{e}_\phi$ are unit vectors in the θ, ϕ directions and F_θ, F_ϕ are the components of the far-zone field amplitude. In terms of F_θ and F_ϕ the Stokes parameters of the far-zone field are

$$S_o = \frac{1}{16\pi^2 r^2}(F_\theta F_\theta^* + F_\phi F_\phi^*) \ , \ S_1 = \frac{1}{16\pi^2 r^2}(F_\theta F_\theta^* - F_\phi F_\phi^*)$$

$$S_2 = \frac{1}{16\pi^2 r^2}(F_\theta F_\phi^* + F_\phi F_\theta^*) \ , \ S_3 = \frac{i}{16\pi^2 r^2}(F_\phi F_\theta^* - F_\theta F_\phi^*) \ .$$

These parameters completely describe the polarization state of the far-zone field [1]. The purpose of the present paper is to report certain global properties the above Stokes possess over the surface of a far-zone sphere.

For S_o we have the following theorems:

Theorem I: S_o can not be constant over a far-zone sphere [2,3].

Theorem II: S_o can be determined over the entire far-zone sphere if S_o is known over a patch. This follows from the fact that F_θ and F_ϕ are entire functions of the complex variables θ and ϕ.

With the aid of Theorems I and II we see that S_o can not be constant even over a patch on the far-zone sphere for if it were constant over a patch it would be constant over the entire sphere.

Theorem III: The locus of S_o = constant must be one or more closed curves or one or more isolated points on the far-zone sphere.

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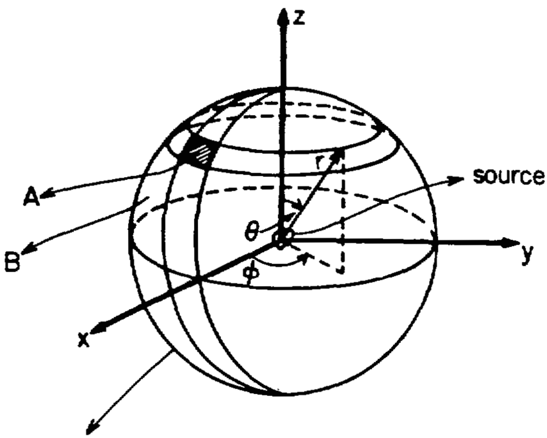

Fig. 1. A is a patch on the surface of a far-zone sphere. B is its complement i.e. A+B = the entire surface of the sphere. If $S_i (i = 0,1,2,3)$ is known over region A, it can be extrapolated to all points of region B. However, if region A reduces to a curve segment, none of the four parameters can be extrapolated.

For S_1, S_2, S_3 we have

Theorem IV: If $S_i (i = 1,2,3)$ is constant over a patch, it must be constant over the entire sphere. However, since $S_0^2 = S_1^2 + S_2^2 + S_3^2$ for monochromatic radiation [4] and since, as mentioned above, S_0 can not be constant over a patch, only two of the three parameters S_1, S_2, S_3 can be constant over a patch at the same time.

Theorem V: The locus of constant $S_i (i = 1,2,3)$ must be one or more isolated points, one or more closed curves, or the entire sphere.

Theorem VI: From a knowledge of $S_i (i = 1,2,3)$ over a patch its value over the entire sphere can be found by extrapolation, as in the case of S_0.

Wherever we have $S_3 = o$ on a far-zone sphere, the polarization is linear. This corresponds to what Müller calls a L-point [5]. Wherever we have $S_1 = S_2 = o$ on a far-zone sphere the polarization is circular. This is what Müller calls a C-point. Müller has shown that if all points of a patch are L-points, then all points of the entire sphere must be L-points. Clearly, this is a special case of our Theorem IV. He has also shown that if all points of a patch are C-points, then all points of the entire sphere must be C-points. This is also a special case of our Theorem IV.

In the design of antenna and radar systems the above theorems delimit the distribution of polarization over a far-zone sphere so that once the polarization over a patch has been specified the polarization over the rest of the sphere is fixed. For example, if at some point P on the far-zone sphere we have left-handed polarization and if at some other point Q on the far-zone sphere we have right-handed polarization, then along any path on the far-zone sphere that connects P and Q at least at one point along the path the polarization is unavoidably linear.

REFERENCES

[1] G. G. Stokes, "On the Composition and Resolution of Streams of Polarized Light from Different Sources," Trans. Cambridge Phil. Soc., 9:399 (1852). Reprinted in his "Mathematical and Physical Papers," vol. III, pp. 233-258, Cambridge University Press, London.

[2] H. F. Mathis, "A Short Proof that an Isotropic Antenna is Impossible," Proc. IRE, 39, 970 (1951).

[3] C. J. Bouwkamp, H. B. G. Casimir, "On Multipole Expansions in the Theory of Electromagnetic Radiation," Physica, 20, 539 (1954).

[4] C. H. Papas, Theory of Electromagnetic Wave Propagation, McGraw-Hill Book Company, New York (1965).

[5] C. Müller, Foundations of the Mathematical Theory of Electromagnetic Waves, Springer-Verlag, New York (1969).